A fuzzy constraint satisfaction approach for signal abstraction

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Abstract

We present the Multivariable Fuzzy Temporal Profile model (MFTP), a formal model which allows us to represent signal patterns and identify their occurrences over the temporal evolution of a set of physical parameters. The pattern comprises a set of findings, each one of which may, in turn, be a pattern, so that its recognition is organized into a hierarchy of abstraction levels, and ultimately they are associated with the appearance of certain distinctive morphologies -profiles- over each parameter. The patterns definition is obtained directly from humans experts,
either with the help of a formal language or with a visual tool developed for this purpose.

The model is based, on one hand, on Fuzzy Set Theory, which allows the vagueness and imprecision which are characteristic of human knowledge to be modelled; and on the other, on the formalism of Constraint Satisfaction Problems (CSP), in order to obtain a representation capable of explicitly capturing the hierarchy of abstraction levels into which the recognition task is organized. We supply algorithms for analyzing the consistency of the information defined by the MFTP, and for the recognition of patterns over signal recordings.

1 Introduction

The proliferation of new, more sophisticated acquisition devices, and the advances made in information and communication technologies, have resulted in a huge volume of data being available to systems supervisors. These data must allow them to understand the internal logic of the system’s operation, the multiple processes concurring in the observations, and the relations that the system establishes with its environment; i.e., the data must permit the interpretation of the system.

This work is based on the premise that interpretation processes are carried out in hierarchically ordered levels of representation. The lower level is made up by the raw data obtained directly from the system. Starting from this level, items of information are aggregated by means of a set of knowledge-driven operations of a heuristic nature. These operations progressively eliminate that part of the available information which is not relevant for the interpretation task in process, generating a representation organized into a set of abstraction levels. Each of the abstraction level supposes an operation of synthesis over the previous level, resulting in a reduction of the volume of data to be
handled and in an increment in their semantics, i.e., their information content [18, 25].

When the object of an abstraction operation evolves over time, temporal abstraction poses specific challenges [2, 25]: the characteristics of the system change over time -and thus its interpretation-, and in order to obtain this interpretation it is necessary to reason over the system’s temporal evolution. Patient supervision, the control of industrial processes, telecommunications network monitoring, etc., are domains in which temporal abstraction operations take on particular importance. In these domains, the volume of data to be handled often exceeds the cognitive capabilities of system supervisors, forcing them to ignore that portion of the data which takes them outside their scope of competence, and this may lead them to make mistakes [19]. The situation is compounded by the fact that data which evolve over time generally correspond to processes requiring rapid response. It is necessary to supply automated solutions to relieve experts of part of the task of interpreting data arising from the observation of the system, in order to allow them to focus more of their efforts on guiding the system to the desired state.

The current work aims to supply a solution for the knowledge-based temporal abstraction of information coming from the temporal evolution of a set of parameters representing a physical system. The starting point is a description of a pattern of interest, which is obtained directly from the experts, either by means of a formal language, or with a visual tool which has been developed for this purpose. The MFTP model is based on the formalism of Constraint Satisfaction Problems (CSP) and on Fuzzy Set Theory; the former supplies a representational structure, facilitating the computational projection of the description made by the expert, while the latter allows the vagueness and imprecision that are characteristic of expert knowledge to be handled. The model supplies a set of algorithms that analyze information consistency, warning ex-
perts of the presence of any inconsistent information, in order for it to be corrected.
Once pattern representation is consistent, pattern recognition is carried out over a sig-
nal recording, using a set of algorithms that attempt to reproduce the set of abstraction
operations that the human experts themselves use in this process (see Fig. 1).

This paper starts with a brief review of previous work. Section III introduces an
example of a fictitious pattern, which we use both to illustrate the basic concepts of
the model (Section IV) and to introduce the problem of analyzing the consistency of
the information (Section V). In Section VI we outline a procedure for recognizing an
MFTP over a signal recording, along with heuristics to speed up the recognition process
and, we briefly present those real applications that have been tackled with the MFTP
model. Finally, we discuss the most salient aspects of this model, and present some
conclusions on it.

2 Previous work

In the bibliography there are a number of temporal abstraction techniques with a set
of features in common: they take an expert’s description as a starting point, deal with
the vagueness and uncertainty of this description, and respond to the variability and
imprecision that are characteristic of the analysis of physical parameters.

The first works appearing in the bibliography deal with signal abstraction by means
of a qualitative description based on a sign algebra-based representation. Thus Cheung
et al. [3] and Konstantinov et al. [14] develop models that allow the qualitative de-
scription of the temporal evolution of a parameter based fundamentally on the signs of
the slope and curvature of the signal.

Fuzzy Set Theory has proven its capability for dealing with imprecision, vagueness
and uncertainty, providing a formal logical framework. This has served as a stimulus for the appearance of new proposals for representing and reasoning on temporal patterns by means of fuzzy sets.

Thus, for example, Drakopoulos et al. [7] present a pattern recognition model in which each pattern is characterized by a set of fuzzy conditionals describing the temporal evolution of a parameter. A rudimentary language is proposed that the authors themselves acknowledge as the bottleneck in their model. Finally, they propose a learning-based acquisition of knowledge, but no specific solution is provided.

Steimann et al. [27] develop a model that enables the description of simple trends ("a smooth increment") over the temporal evolution of a single parameter based on the concept of fuzzy trajectory. In order to acquire knowledge relating to a trend, a possibility distribution is constructed that is projected over the samples of the evolution, proposing a membership function for each one of them. The fuzzy trend is calculated as the envelope of all the membership functions. The model is aimed basically at detecting linear trends, but not more complex morphologies.

Lowe et al. [15] extend the previous model to allow the detection of multivariable patterns; nevertheless, its weak point is still the modelling of profiles of more than one trend. This proposal does not provide a solution for acquiring the patterns. The expert describes patterns using natural language, and from these descriptions the knowledge engineer derives the parameters of the model and implements the matching algorithms.

Félix et al. [10] propose the Fuzzy Temporal Profile (FTP) model for the recognition and representation of morphologies -not just linear trends- over the evolution of a physical parameter. This proposal focuses on the study of how a fuzzy constraint network can model those sentences from natural language that describe evolution of a single parameter.
Milios et al. [18] do not propose a specific temporal abstraction technique, but an organization into multiple levels of abstraction in the application of signal processing software, stressing its advantages as a means of structuring signal information and explicitly representing heuristic knowledge associated with signals and signal processing algorithms.

Medicine, where a large part of the knowledge available is heuristic and based on experience, has been the principal application domain for abstraction techniques [2, 6, 7, 15]. The control of industrial processes [3, 14, 18] and the control of land traffic [26] are other domains in which they have also been applied.

MFTP model is an extension of the FTP model with ideas borrowed from Lowe et al. [15] and from Milios et al. [18]. From the former it incorporates the idea of defining patterns over the temporal evolution of several parameters in order to identify a set of events which, taken in isolation, may be irrelevant, but if combined are of interest for the experts on the application domain. From the latter the MFTP model takes the organization of the different pieces which make up the pattern in a hierarchy of levels of abstraction, providing support for the representation of the heuristic knowledge-driven operations that experts use when they identify the pattern themselves.

3 An example

To illustrate certain features of our proposal we will punctually use real examples taken from the domain applications where the MFTP model has been applied: patient monitoring [23] and mobile robotics [24]. However, given that no single real pattern we have encountered in these domains allow us to illustrate all the capabilities of the model, here we present an artificial example, which is more suitable to illustrate the model’s
expressive power and which will be modelled in detail in the article.

An employee is charged with the supervision of a set of parameters arising from the monitoring of an industrial process. Among the instructions he receives are the following (see Fig. 2):

“The most common malfunction pattern in the system is as follows: parameter 1 \((P^1)\) increases from a low value to approximately fifty units, and subsequently falls sharply by 10 units in no more than 20 seconds. At approximately the same time as the onset of the sharp fall in \(P^1\), parameter 2 \((P^2)\) starts to rise moderately. \(P^2\) continues to rise until approximately 10 seconds after the sharp fall in \(P^1\) has come to an end, having increased approximately 40 units. Its magnitude then remains constant for approximately half a minute, and then falls around 30 units at a rate of approximately 4 units per second in a little over 10 seconds. The senior plant manager must be notified immediately of any occurrence of this pattern (which we shall call \(MF_1\)).

Another abnormal function pattern arises when there is a moderate increase of around 20 units in a little over 50 seconds in parameter 3 \((P^3)\), with a subsequent sharp rise of almost 40 units in no more than 10 seconds. If almost simultaneously with the onset of the sharp rise in \(P^3\), parameter 4 \((P^4)\) starts to rise, reaching a value approximately 30 units higher than the maximum value reached for 3 in less than two minutes, the plant manager must be notified immediately of any occurrence of this pattern (which we shall call \(MF_2\)).

When both \(MF_1\) and \(MF_2\) occur, and the onset of the rise in \(P^2\) takes place between 15 and 30 seconds before the onset of the sharp rise in \(P^3\),
and \( P^4 \) has reached its maximum value before \( P^2 \) has finished falling, 
\( MF'2 \) has occurred as a consequence of \( MF1 \), and there has been an irreversibl
fault in the system. In this case, the process must be halted immediately and, subsequently, the plant manager notified.”

4 Prior definitions

In this section we introduce some notions upon which the MFTP model is based. We consider time as being projected onto a one-dimensional discrete axis \( \tau = \{t_0, t_1, ..., t_i, ...\} \). Thus given an \( i \) belonging to the set of natural numbers \( \mathbb{N} \), \( t_i \) represents a precise instant. We assume that \( t_0 \) represents the temporal origin, before which the existence of any fact is not relevant for the problem under consideration. We consider a total order relation between them, in such a way that for every \( i \in \mathbb{N} \), \( t_{i+1} - t_i = \Delta t \), where \( \Delta t \) is a constant. \( \Delta t \) is the minimum step of the temporal axis, i.e., the sampling period.

Given as discourse universe the set of real numbers \( \mathbb{R} \), a fuzzy number \( A \) is a normal (\( \exists v \in \mathbb{R}, \mu_A(v) = 1 \)) and convex (\( \forall v, v', v'' \in \mathbb{R}, v' \in [v, v''], \mu_A(v') \geq \min \{\mu_A(v), \mu_A(v'')\} \)) fuzzy subset of \( \mathbb{R} \). We obtain a fuzzy number \( A \) from a flexible constraint given by a possibility distribution \( \pi_A \), which defines a mapping from \( \mathbb{R} \) to the real interval \([0, 1]\). Given a precise number \( v \in \mathbb{R}, \pi_A(v) \in [0, 1] \) represents the possibility of \( A \) being precisely \( v \). By means of \( \pi_A \) we define a fuzzy subset \( A \) of \( \mathbb{R} \), which contains the possible values of \( A \), where \( A \) is a disjoint subset, in the sense that its elements represent mutually excluding alternatives for \( A \).

We represent a fuzzy number by means of a trapezoid \( A = (\alpha, \beta, \gamma, \delta) \), \( \alpha \leq \beta \leq \gamma \leq \delta \), where \([\beta, \gamma]\) represents the core, \( \text{core}(A) = \{v \in \mathbb{R}|\pi_A(v) = 1\} \), and \([\alpha, \delta[ \)
represents the support, \( \text{supp}(A) = \{ v \in \mathbb{R} | \pi_A(v) > 0 \} \). Although the validity of MFTP model is not restricted to a specific representation of possibility distributions, in practice it will suffice to work with trapezoidal distributions, given that in possibility theory, what matters is the order of the possibility degrees attached to the different values of the universe of discourse, rather than the precise assignment of possibility degrees [9].

Adhering to Zadeh’s extension principle [28], we define the sum \((\oplus)\), subtraction \((\ominus)\), multiplication \((\otimes)\) and division \((\oslash)\) of two fuzzy numbers \(A\) and \(B\) as the fuzzy number \(A \oplus B\) given by:

\[
\forall A, B \subset \mathbb{R}, \forall x, y, z \in \mathbb{R}, \pi_{A \oplus B}(z) = \max_{z = x \ast y} \min \{ \pi_A(x), \pi_B(y) \}
\]

Where \(\ast\) represents the corresponding classic arithmetic operation. It is easily proved that the addition, the subtraction and the product of two fuzzy numbers is a fuzzy number, and that they are associative and commutative operations. The product is not distributive with respect to addition (it is only so in \(\mathbb{R}^+\)) [13]. Division is defined between fuzzy numbers included only on a semi-axis of the real line. This can be easily extended by dividing separately the positive parts of \(A\) and \(B\), and the symmetrical ones of their negative parts, to calculate the positive part of \(A \oslash B\), and dividing the opposite parts of \(A\) and \(B\), to calculate the negative part of \(A \oslash B\) [20].

Fuzzy subtraction allows us to define the concept of fuzzy increment in order to represent extensions of intervals or, in general the difference between two numbers. A fuzzy increment \(I\) is represented by means of a fuzzy number. In this way, given an \(i \in \mathbb{R}, \pi_D(i) \in [0, 1]\) represents the possibility of \(I\) being precisely equal to \(i\). Given an ordered pair of fuzzy numbers \((A, B)\), we can talk of a fuzzy increment given by the subtraction \(I = B \ominus A\).
The MFTP model

The Multivariable Fuzzy Temporal Profile (MFTP) model enables the representation and identification of a pattern $\mathcal{M}$ of special significance over the temporal evolution of a set of parameters $\mathcal{P} = \{P^1, ..., P^z\}$, where each parameter $P^p$ is obtained by means of an acquisition and sampling process: $P^p = \{(v^p_{[s]}, t^p_{[s]})|s \in \mathbb{N}\}$. This pattern is described by a human expert and consists of a set of findings and relations between them. In our example, $\mathcal{P} = \{P^1, P^2, P^3, P^4\}$.

The MFTP model is an extension of the Fuzzy Temporal Profile (FTP) model [20], which enables the representation and identification of a signal pattern over a single physical parameter. The MFTP model takes an important step forward. Each profile can be aggregated as a piece of a more complex pattern over multiple parameters. Each pattern defines a finding of interest in terms of the interpretation of the system behaviour. The ability to relate the occurrence of different findings among parameters is of great importance, as often the appearance of a finding over a single parameter, which on its own may not be a major determinant, may well be of interest if it appears to be related with other findings on other parameters, which do not seem to be definitive either when considered in isolation.

In our example, the appearance of the pattern described over $P^3$ is not associated with the occurrence of $MF_2$ if $P^4$ does not present a sharp rise. In a similar way, in the medical domain a slight but sustained increase in heart rate in a patient may be provoked by many innocuous causes. However if this increase is simultaneous with a light, sustained decrease in the systolic blood pressure, which is also irrelevant taken on its own, it may signal the start of a life threatening pathology: hypovolemia.

The MFTP model is based on CSP formalism and on Fuzzy Set Theory. An MFTP represents a pattern by means of a network of fuzzy constraints between a set of signif-
significant points (points with a special relevance for the expert) defined over the temporal evolution of the system.

Definition 1 We define a significant point \( X_p^i = < V_p^i, T_p^i > \) on a physical parameter \( P_p \), as the pair formed by a variable from the domain \( V_p^i \) and a temporal variable \( T_p^i \). In the absence of any constraints, \( V_p^i \) and \( T_p^i \) may take any precise value \( v_p^i \) and \( t_p^i \), respectively.

In this work we shall use the notation \((v_p^i[s_i], t_p^i[s_i])\) to refer to any sample of the parameter \( P_p \), and \((v_p^i, t_p^i)\) to represent a sample of the parameter \( P_p \) which has been assigned to the significant point \( X_p^i \) of some MFTP.

We will define a set of fuzzy constraints between the variables of the significant points, providing a computable support for soft descriptions of the shape of a pattern. Given that the knowledge projected in the model is obtained directly from humans, it is usually of a descriptive nature. In this sense, experience has shown that in order to describe the temporal evolution of a set of parameters, a set of constraints limiting the fuzzy temporal extension, fuzzy increment and fuzzy slope between a set of significant points captures a good number of nuances.

Definition 2 We define a binary constraint \( L_{pq}^{ij} \) on two temporal variables \( T_p^i \) and \( T_q^j \) by means of a normal and convex possibility distribution \( \pi_{L_{pq}^{ij}}(l) \) over \( \mathbb{Z} \), such that \( \forall l \in \mathbb{Z} : \pi_{L_{pq}^{ij}}(l) \in [0, 1] \). Given a precise value \( t_{ij}^{pq} \), \( \pi_{L_{pq}^{ij}}(t_{ij}^{pq}) \) represents the possibility of the temporal distance between \( T_p^i \) and \( T_q^j \) taking the value \( t_{ij}^{pq} \).

In the absence of other constraints, the assignments \( T_p^i = t_p^i \) and \( T_q^j = t_q^j \) are possible if \( \pi_{L_{pq}^{ij}}(t_q^j - t_p^i) > 0 \) is satisfied. The constraints \( L_{pq}^{ij} \) allow linguistic descriptions that limit the fuzzy temporal distance between a pair of significant points to be modelled. Formally, \( \pi_{L_{pq}^{ij}} \) corresponds to the possibility distribution of a fuzzy increment.
on the temporal domain, i.e., a fuzzy duration or fuzzy temporal extension. Thus, a binary constraint $L_{ij}^{pq}$ can be interpreted as the assignment of a fuzzy temporal duration to the temporal distance between the variables $T_i^p$ and $T_j^q$. If both variables are defined over the temporal evolution of one single parameter ($p = q$), $L_{ij}^p$ represents the temporal extension during which the value or rate of change of the parameter remains constant. For example, in Fig. 2, $L_{23}^1$ models the linguistic description “no more than 20 seconds”. It is required of the constraints $L_{ij}^p$, $i > j$, that $L_{ij}^p > 0$. In this way, we avoid a parameter taking two different values at the same time instant. When $T_i^p$ and $T_j^q$ are defined over different parameters ($p \neq q$), $L_{ij}^{pq}$ makes it possible to describe the temporal layout of two findings defined over these parameters. For example, in Fig. 2 $L_{21}^{12}$ models the linguistic description “at approximately the same time as the onset of the sharp fall in $P_1$, $P_2$ starts to rise moderately”.

**Definition 3** We define a binary constraint $D_{ij}^{pq}$ on two variables of the domain $V_i^p$ and $V_j^q$, in a similar way to the constraint $L_{ij}^{pq}$, by means of a normal and convex possibility distribution $\pi_{D_{ij}^{pq}}(d)$ over $\mathbb{R}$, such that $\forall d \in \mathbb{R} : \pi_{D_{ij}^{pq}}(d) \in [0, 1]$. Given a precise value $d_{ij}^{pq}$, $\pi_{D_{ij}^{pq}}(d_{ij}^{pq})$ represents the possibility of the difference between $V_i^p$ and $V_j^q$ taking the value $d_{ij}^{pq}$.

In the absence of other constraints, the assignments $V_i^p = v_i^p$ and $V_j^q = v_j^q$ are possible if $\pi_{D_{ij}^{pq}}(v_j^q - v_i^p) > 0$ is satisfied. With the constraints $D_{ij}^{pq}$ it is possible to model linguistic descriptions that limit the difference in value between a pair of significant points. Formally, $\pi_{D_{ij}^{pq}}$ corresponds to the possibility distribution of a fuzzy increment. Thus, a binary constraint $D_{ij}^{pq}$ can be interpreted as the assignment of a fuzzy increment to the difference in magnitude between the variables $V_i^p$ and $V_j^q$. If both variables are defined over the temporal evolution of one single parameter ($p = q$), $D_{ij}^p$ represents the increase in value that this parameter undergoes between two significant points. For ex-
ample, in Fig. 2, $D_{23}^1$ models the description “falls sharply by almost 10 units”. When they are defined between significant points belonging to different but commensurable parameters ($p \neq q$), they permit the description of relations between the magnitudes of both parameters. For example in Fig. 2, $D_{43}^{43}$ “$P^4$ starts to rise reaching a value approximately 30 units higher than the maximum value reached for $P^3$.”

Following the bibliography on constraint networks [5], and with the aim of obtaining a more compact notation, we define the origin significant point $X_p^0 = \langle V_p^0, T_p^0 \rangle$ which will make it possible to represent temporal constraints (e.g. “a little after 22:30”) and value constraints (e.g. “approximately 15 units”) as constraints of duration and increment, respectively, relating to the origin significant point. Any arbitrary value can be assigned to $X_p^0$, although it is habitually assigned the value $V_p^0 = 0, T_p^0 = 0$. In Fig. 2 $D_{0_1}^{1}$ and $D_{0_2}^{1}$ model the linguistic description “$P^1$ increases from a low value” and “to approximately 50 units”, respectively.

**Definition 4** We define a quaternary constraint $M_{ij}^p$ on two significant points $X_i^p$ and $X_j^p$ by means of a normal and convex possibility distribution $\pi_{M_{ij}^p}(m)$ over $\mathbb{R}$, such that $\forall m \in \mathbb{R} : \pi_{M_{ij}^p}(m) \in [0, 1]$. Given a precise value $m_{ij}^p$, $\pi_{M_{ij}^p}(m_{ij}^p)$ represents the possibility of the slope of the line that joins $X_i^p$ and $X_j^p$ taking the value $m_{ij}^p$.

The constraint $M_{ij}^p$ jointly restricts the domains of $V_i^p$, $V_j^p$, $T_i^p$ and $T_j^p$. In the absence of other constraints, the assignments $V_i^p = v_i^p, V_j^p = v_j^p, T_i^p = t_i^p$ and $T_j^p = t_j^p$ are possible if $\pi_{M_{ij}^p}((v_j^p - v_i^p)/(t_j^p - t_i^p)) > 0$ is satisfied. With the constraints $M_{ij}^p$ it is possible to model linguistic descriptions of the rate of change of a parameter. For example, in Fig. 2 $M_{34}^2$ models the description “... then falls around 30 units at a rate of approximately 4 units per second”. Formally, $\pi_{M_{ij}^p}$ corresponds to the possibility distribution of a fuzzy number, which we can interpret as the fuzzy slope of the line which joins $X_i^p$ and $X_j^p$. 

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With these elements, we can describe a pattern over the temporal evolution of a set of parameters as a network of fuzzy constraints $R_{pq}^{ij}$ between pairs of significant points $X_p^i$ and $X_q^j$. On one hand, this network allows the morphology of a signal finding to be described, by means of constraints over points belonging to the same parameter ($p = q$), which define a joint constraint between these points, taking the form $R_{pq}^{ij} \equiv R_{ij}^p = <L_{ij}^p, D_{ij}^p, M_{ij}^p>$. On the other hand, the relative layout between the different findings that form part of a pattern can be described by means of constraints between points belonging to different parameters ($p \neq q$), which define a joint constraint $R_{pq}^{ij} = <L_{ij}^{pq}, D_{ij}^{pq}>$.

**Definition 5** We define a **Multivariable Fuzzy Temporal Profile (MFTP)** $\mathcal{M} = <W, X, R>$ as a finite set of MFTPs $W = \{\mathcal{M}_1, ..., \mathcal{M}_n\}$, a finite set of significant points $X = \{X_p^i : 0 \leq i \leq n_p, 1 \leq p \leq z\}$ and a finite set of constraints $R = \{R_{pq}^{ij} : 1 \leq p, q \leq z, 0 \leq i \leq n_p, 0 \leq j \leq n_q\}$ between points of $W$ and $X$.

$z$ is the number of parameters involved in $\mathcal{M}$; $n_p$ is the number of significant points defined over the parameter $P^p$; and $n_q$ is the number of significant points defined over the parameter $P^q$. The constraints $R_{pq}^{ij} \in R$ can be defined between significant points belonging to $X$, between significant points belonging to the set of subMFTPs $W$ or between both types of significant points. When a constraint is not specified, it is understood that any value is completely possible, and it is called **universal constraint**, $\pi_{U}(x) = 1, \forall x \in \mathbb{R}$. Given a constraint $R_{pq}^{ij}$ its symmetrical constraint $R_{pq}^{ji}$ is defined by $L_{ji}^{pq} = -L_{ij}^{pq}$, $D_{ij}^{pq} = -D_{ij}^{pq}$ and, when $p = q$, $M_{ij}^q = M_{ij}^p$. $R_{pq}^{ij}$ and $R_{pq}^{ji}$ contain the same information, i.e., they are redundant. We suppose that any constraint of the form $L_{ij}^p(D_{ij}^p)$ is equivalent to the crisp set $\{0\}$; i.e., $\pi_{L_{ij}^p}(0) = \pi_{D_{ij}^p}(0) = 1$, and $\forall l \neq 0, \pi_{L_{ij}^p}(l) = 0$ and $\forall d \neq 0, \pi_{D_{ij}^p}(d) = 0$. We suppose that the constraint $M_{ij}^p$ is
equivalent to the universal constraint.

An MFTP can be represented by a graph in which nodes correspond to significant points, and arcs correspond to constraints (see Fig. 2).

The recursive structure of the MFTP model is inspired by the way that humans define patterns; a complex pattern is often made up of a set of findings and a set of relations between them. Each of the findings of the pattern may also be a pattern, and may comprise a set of findings and relations between them, and so on, successively.

For instance, the “irreversible fault” pattern is made up of two findings for which certain temporal relation between them must be satisfied: $\mathcal{M}^{IF} = \langle \mathcal{M}^{MF_1}, \mathcal{M}^{MF_2}, \emptyset, \{L^{23}_{12}, L^{23}_{23}\} \rangle$. Each of these findings - which represent each of the malfunctioning patterns - is in turn an MFTP that is made up of two findings which must satisfy certain temporal and magnitude relations between them: $\mathcal{M}^{MF_1} = \langle \mathcal{M}^{PF_1}, \mathcal{M}^{PF_2}, \emptyset, \{L^{23}_{12}\} \rangle$ and $\mathcal{M}^{MF_2} = \langle \mathcal{M}^{PF_3}, \mathcal{M}^{PF_4}, \emptyset, \{L^{23}_{23}\} \rangle$. And the findings which make up $MF_1$ and $MF_2$ also are MFTPs which represent a morphology defined over the temporal evolution of one of the system’s parameters: $\mathcal{M}^{PF_1} = \langle \emptyset, \{X^1_1, X^1_2, X^1_3\}, \{D^1_{01}, D^1_{02}, R^1_{12}, R^1_{23}\} \rangle$, $\mathcal{M}^{PF_2} = \langle \emptyset, \{X^2_1, X^2_2, X^2_3, X^2_4\}, \{R^2_{12}, R^2_{23}, R^2_{34}, R^2_{14}\} \rangle$, $\mathcal{M}^{PF_3} = \langle \emptyset, \{X^3_1, X^3_2, X^3_3\}, \{R^3_{12}, R^3_{23}\} \rangle$, $\mathcal{M}^{PF_4} = \langle \emptyset, \{X^4_1, X^4_2\}, \{R^4_{12}, R^4_{23}\} \rangle$.

The hierarchical organization facilitates the elicitation and maintenance of knowledge, since the computational representation of the pattern is closer to the mental model that experts have of it. Each finding on each abstraction level that the expert uses to reason over the system is represented by an MFTP. The abstraction operations that project a set of findings (each one of which is represented by a subMFTP $\mathcal{M}_k \in \mathcal{W}^{M_k}$) over a finding from a higher level, $\mathcal{M}$, are modelled by means of a set of relations that the subMFTPs must satisfy in order to give rise to $\mathcal{M}$. The power of the constraint networks formalism for representing information of hierarchical nature has already been
proven in the bibliography [12, 16].

By explicitly capturing those abstraction operations that lead to an identification of a pattern, it is possible to give detailed explanations on the results of the recognition. Thus when the pattern is identified over the evolution of the system $\mathcal{S}$, we will be able to reconstruct all the operations that have made it possible to reach the pattern from the sampled signal; when it is not identified, we will be able to say which abstraction operations have failed.

The hierarchical organization of information endows the model with modularity, allowing the same finding to form part of a number of MFTPs. In our example, the pattern $\mathcal{M}_1^{\text{P1}}$ belongs both to $\mathcal{M}_1^{\text{MF1}}$ and to $\mathcal{M}_1^{\text{IF}}$. In the medical domain, a sustained slight increase for approximately 30 seconds in the heart rate constitutes a finding that belongs to the pattern that, at the signal level, characterizes both a pulmonary embolism and a hypovolemia.

The modularity simplifies the implementation of the recognition algorithms by means of a multiagent architecture, in which each agent is responsible for the recognition of one of the MFTPs, and where different patterns which have findings in common may share information. Thus for example, in a patient monitoring system watching for occurrences of pulmonary embolisms and hypovolemia, in order to identify the pattern representing the increases in heart rate $\mathcal{M}_1^{\text{Slight increase FC}}$, only one agent will be needed; the information it generates will be reused by both the agent responsible for detecting $\mathcal{M}_1^{\text{Embolism}}$ and the one responsible for detecting $\mathcal{M}_1^{\text{Hypovolemia}}$.

5.1 Solution of a MFTP

The ultimate aim of the MFTP model is to supply tools for pattern recognition and temporal abstraction in systems characterized by the temporal evolution of a set of pa-
rameters. Once this problem has been formulated as a CSP, the recognition task is resolved by searching for solutions to the CSP which define $\mathcal{M}$; i.e., sets of assignments to all those significant points belonging to $\mathcal{M}$ that are compatible with the constraints described by the expert.

**Definition 6** $A^p_i$ denotes the assignment of a sample $(v^p_{[a]}, t^p_{[a]})$ of the evolution of the parameter $P^p$ to the significant point $X^p_i$: i.e., $A^p_i = (v^p_i, t^p_i) = (v^p_{[a]}, t^p_{[a]})$ signifies that $V^p_i = v^p_i = v^p_{[a]}$ and $T^p_i = t^p_i = t^p_{[a]}$.

A pair of assignments $(A^p_i, A^q_j)$ induces three partial assignments: $t^p_{ij} = (t^p_i - t^p_j)$, $d^p_{ij} = (v^p_j - v^p_i)$ and, in the case that $p = q$, $m^p_{ij} = (v^p_j - v^p_i)/(t^p_j - t^p_i)$. We say that a pair of assignments $(A^p_i, A^q_j)$ is valid under constraint $R^pq_{ij}$, if and only if it satisfies the constraints that exist between the significant points $X^p_i$ and $X^q_j$: $\pi_{L^pq_{ij}}(I^pq_{ij}) > 0$, $\pi_{D^pq_{ij}}(d^pq_{ij}) > 0$ and, in case that $p = q$, $\pi_{M^pq_{ij}}(m^pq_{ij}) > 0$. Making use of the min operation to model the conjunctive combination of constraints, the degree of possibility of a pair of assignments being valid is given by $\pi_{I^pq_{ij}}(A^p_i, A^q_j) = \min\{\pi_{L^pq_{ij}}(I^pq_{ij}), \pi_{D^pq_{ij}}(d^pq_{ij}), \pi_{M^pq_{ij}}(m^pq_{ij})\}$, if $p = q$, and by $\pi_{I^pq_{ij}}(A_i, A_j) = \min\{\pi_{L^pq_{ij}}(I^pq_{ij}), \pi_{D^pq_{ij}}(d^pq_{ij})\}$, if $p \neq q$. We say that a constraint $R^pq_{ij}$ is satisfiable if there is at least one pair of assignments which satisfy the constraint with degree 1.

**Definition 7** We say that a set of assignments to all the significant points of $\mathcal{M}$, $A = \{A^1_i, \ldots, A_{n^1_i}, \ldots, A^1_{z_i}, \ldots, A^z_{n_i}\}$, is a $\sigma$-solution of $\mathcal{M}$ if and only if it satisfies the set of constraints that make up $\mathcal{M}$ with degree $\sigma$:

$$\pi_{\mathcal{M}}(A) = \min\{\min_{M_h \in \mathcal{W}^M} \{\pi_{M_h}(A^{M_h})\}, \min_{R^pq_{ij} \in \mathcal{R}^M} \{\pi_{I^pq_{ij}}(A^p_i, A^q_j)\}\} = \sigma$$

where $1 \leq p, q \leq z$, $0 \leq i \leq n^p$ and $0 \leq j \leq n^q$; $A^{M_h}$ is the projection of $A$ over the set of significant points that belong to $\mathcal{M}_h$, i.e., $\forall A^p_i \in A; A^p_i \in A^{M_h} \iff$
$X_i^p \in X^{\mathcal{M}_h}$. $A_i^p$ and $A_j^q$ are the assignment that $\mathcal{A}$ makes to the significant points $X_i^p$ and $X_j^q$, respectively. $\pi^{\mathcal{M}_h}$ is the degree of satisfaction of the finding $\mathcal{M}_h$. $\pi^{\mathcal{M}}(\mathcal{A})$ represents the degree of similarity between a fragment of the evolution of $\mathcal{P}$ with the pattern of findings $\mathcal{M}$.

We say that an MFTP $\mathcal{M}$ is **satisfiable** if there exists at least one $\sigma$-solution with $\sigma = 1$. An MFTP represents a pattern that may occur over the temporal evolution of a physical system; hence, although it may be possible to find solutions which do not coincide completely with its description, there must always be at least one prototype of the pattern, that is, a completely possible solution.

### 6 Pattern consistency analysis

Expert knowledge projected onto an MFTP is obtained in a descriptive manner. Knowledge thus described intrinsically contains redundant information, which is incorporated by the model into its representation. Let us consider, for example, that information concerning the rate of change of a parameter is redundant with regard to the increase in magnitude and the temporal interval in which it takes place. This redundancy supplies a great deal of flexibility in knowledge acquisition, since it allows the same information to be represented in different ways.

The downside of this expressive power is that contradictory, or unnecessarily imprecise information may be incorporated into the representation. If the pattern contains contradictory information, it will not be possible to find solutions, and hence it makes no sense to embark on the recognition phase; in this case we shall say that the pattern is **inconsistent**. If the projected information is not as precise as possible, the recognition of the pattern over the temporal evolution of the system will not be as efficient as it
could be; in this case we say that the pattern is not minimal. The pattern is minimal if its network $\mathcal{M}$ is that network in which each of its constraints is more precise than any other constraint that defines the same pattern.

We call the task of detecting inconsistencies and refining the information projected onto an MFTP the consistency analysis of the pattern. This task is carried out by means of an inference mechanism for rendering the implicit information in the constraint network explicit: the propagation of constraints. If any contradictory information is identified during the consistency analysis process, we will have discovered an inconsistency in the definition of the MFTP, and its knowledge must be revised; otherwise, we may proceed with the recognition phase (see Fig. 1).

The constraints $L^p_{ij}$, $D^p_{ij}$ and $M^p_{ij}$, between a pair of significant points $X^p_i$ and $X^p_j$, are redundant. This redundancy can be used to reduce their imprecision. Thus, for example, the linguistic description “and then falls around 40 units ... at a rate of approximately 4 units per second in a little over 10 seconds” can be projected over the constraints: $L^2_{34} = \text{“a little over 10 seconds”} = (10, 11, 13, 15)$, $D^2_{34} = \text{“falls around 40 units”} = (32, 36, 44, 46)$, and $M^2_{34} = \text{“rate of approximately 4 units per second”} = (3.2, 3.5, 4.5, 4.9)$. The constraints $L^2_{34}$ and $D^2_{34}$ define a slope $D^2_{34} \otimes L^2_{34} = (2.1, 2.8, 4, 4.6)$. Thus they supply additional information to that given by $M^2_{34}$. In certain cases, it is possible to infer more precise information by combining all three items of information. We can substitute the initial value of the fuzzy slope with $M^2_{34} \cap (D^2_{34} \otimes L^2_{34}) = (3.2, 3.5, 4, 4.6)$.

We can combine each piece of information from $R^p_{ij} = \langle L^p_{ij}, D^p_{ij}, M^p_{ij} \rangle$ with the others, in order to detect any possible inconsistencies in the constraints and reduce their imprecision. To this end, we define an operator $T : \{R^p_{ij}\} \rightarrow \{Q^p_{ij}\}$, such that:

$$(L^p_{ij})_Q = (L^p_{ij})_R \cap ((D^p_{ij})_R \otimes (M^p_{ij})_R), \quad (D^p_{ij})_Q = (D^p_{ij})_R \cap ((M^p_{ij})_R \otimes (L^p_{ij})_R),$$
and \((M_{ij}^p)_Q = (M_{ij}^p)_R \cap ((D_{ij}^p)_R \ominus (L_{ij}^p)_R)\). It can be demonstrated that given a constraint \(R_{ij}^p\), the constraint \(Q_{ij}^p\), where \(Q_{ij}^p = T(R_{ij}^p)\), is equivalent to the initial one, i.e., \(\forall(A_i^p, A_{ij}^p): \pi_{Q_{ij}^p}(A_i^p, A_{ij}^p) = \pi_{R_{ij}^p}(A_i^p, A_{ij}^p)[20]\). Furthermore, the operator \(T\) is idempotent \((T^2 = T)\), and the constraint \(Q_{ij}^p\) is locally minimal and unique.

Applying the operator \(T\) to all the constraints \(R_{ij}^p\) of an MFTP achieves certain level of local consistency which we shall call section consistency.

An MFTP contains in its representation a Fuzzy Temporal Constraint Network (FCTN) [17]. FCTN is an extension of the Simple Temporal Problem [5], with fuzzy constraints: an FCTN is a CSP involving a set of variables \(T = \{T_0, T_1, \ldots, T_n\}\) defined over the temporal axis, which have continuous domains; and a set of constraints \(C = \{C_{ij}, 0 \leq i, j \leq n\}\), where \(C_{ij}\) is given by a normal and convex possibility distribution \(\pi_{C_{ij}}(l)\). The FCTN contained in the MFTP \(M\) is given by \(M_T = \{T, \mathcal{L}\}\) where \(T = \{T_{ij}^p, 1 \leq p \leq z, 0 \leq i \leq n^p\}\) are the temporal variables of the significant points and \(\mathcal{L} = \{L_{ij}^p, 1 \leq p, q \leq z, 0 \leq i \leq n^p, 0 \leq j \leq n^q\}\) are the fuzzy temporal extension constraints. It can be shown that an algorithm that obtains path consistency in a FCTN, i.e., an algorithm that realizes \(\forall p, q, r\) and \(\forall i, j, k, 1 \leq p, q, r \leq z, 0 \leq i \leq n^p, 0 \leq j \leq n^q, 0 \leq k \leq n^r: L_{ij}^p = L_{ik}^p \cap (L_{kj}^q \oplus L_{kj}^r)\) will obtain an FCTN equivalent to the initial one, and which is minimal [17]. The computational complexity of this process is \(O(n^3)\), where \(n\) is the number of nodes in the network.

The subnetwork of increments of the MFTP, \(M_V = \{\mathcal{V}, \mathcal{D}\}\) where \(\mathcal{V} = \{V_i^p, 1 \leq p \leq z, 0 \leq i \leq n^p\}\) and \(\mathcal{D} = \{D_{ij}^p, 1 \leq p, q \leq z, 0 \leq i \leq n^p, 0 \leq j \leq n^q\}\), is formally (although not semantically) equivalent to a FCTN. Thus a minimal network equivalent to \(M_V\) can be found through the same procedure. However, the fact that \(M_T\) and \(M_V\) are minimal does not mean that \(M\) will be so, as there are quaternary constraints \(M_{ij}^p\) connecting variables of both networks.
Obtaining path consistency on the networks \( M_T \) and \( M_V \) and obtaining section consistency for the network \( M \) achieves certain levels of local consistency in the MFTP network and may identify inconsistencies in its definition. However, it does not obtain the minimal network, rather a covering of it. It has been proved that obtaining a minimal network for an MFTP is NP-hard [20].

6.1 A tractable topology

The most influent factor in the complexity of obtaining the minimal network of a CSP is the topology of the network. Some authors have studied a set of local consistency levels, which, under a predefined assignment order, allows us to obtain solutions without backtracking [4, 11]. We extend these ideas to the MFTP model, defining a topology characterized by a dense \( M_T \) network, i.e., any pair of temporal variables in the MFTP can be connected by a constraint, and a cycle-free network \( M_V \) (see Fig 3). This topology seems to respond adequately to those descriptions from experts in which they use multiple temporal references to introduce each one of the significant points of the pattern, while they describe the value of each point by means of increments with respect to the previous significant point.

**Definition 8** We define a **Sequential-MFTP (S-MFTP)**, and denote by \( M_S = < \emptyset, X^{M_S}, R^{M_S} > \), as a finite set of significant points \( X^{M_S} = \{ X_p^i : 0 \leq i \leq n^p, 1 \leq p \leq z \} \) and a finite set of constraints \( R^{M_S} = \{ < L_{pq}^{ij} >, 1 \leq p, q \leq z, 0 \leq i \leq n^p, 0 \leq j \leq n^q \} \cup \{ < D_{ij}^p, M_{ij}^p >, 1 \leq p \leq z, 1 \leq i < n^p, j = i + 1 \} \cup \{ D_{ij}^{i+1}, ..., D_{ij}^{z+1}, ..., D_{ij}^{z+1}, i \neq 0, j \neq 0 \} \).

In order to assemble solutions without backtracking, the assignment must be carried out following a particular order, denoted by \( o \): starting with any significant point \( X_p^i \); then running through the significant points of the same parameter, first in ascending
order: \{X^{p_{i+1}}_r, ..., X^{p_n}_r\}, and then in descending order: \{X^{p_{i-1}}_r, ..., X^{p_1}_r\}. If there is any significant point \(X^q_j\), \(q \neq p\), which is connected by an increment constraint with a point that has already been assigned, this will be the next significant point. Then the rest of the points defined on the same parameter are run through in the same order as previously described. Otherwise any \(X^q_j\) is selected. This process is repeated until all significant points have been ordered.

When applied to an S-MFTP \(M_S\), the \textsc{sequential}\_\textsc{consistency} procedure (see Fig. 4) returns a new MFTP \(N_S\), which is equivalent to the initial one and for which it is possible to find solutions without backtracking, following the assignment order \(o\), if all the constraints of \(N_S\) are normalized. This algorithm creates a list \(Q\), which is initialized with all those paths whose consistency must be revised. In this algorithm, the tuple \((p, q, r, i, j, k)\) represents the path running through the variables \(T^p_i\), \(T^q_j\) and \(T^r_k\), by means of the constraints \(L^{pr}_{ik}\) and \(L^{rq}_{kj}\). \textsc{revise\_path}(\(p, q, r, i, j, k\)) (see Fig. 5) can modify the constraint \(L^{pq}_{ij}\), therefore it is necessary to analyze its related paths, those which run through the variables \(T^p_i\), \(T^q_j\) and \(T^r_k\) or the variables \(T^r_k\), \(T^p_i\) and \(T^q_j\), where \(1 \leq r \leq z\) and \(1 \leq k \leq n^r\), \(n^r\) being the number of significant points of \(M_S\) defined over the parameter \(P^r\), and \(z\) the number of parameters in the system. The \textsc{related\_paths}(\(p, q, r, i, j, k\)) procedure incorporates these paths into the consistency analysis. The computational complexity of the \textsc{sequential}\_\textsc{consistency} procedure is \(O(n^3)\), where \(n\) is the number of nodes in the network, and is given by the complexity of obtaining path consistency in a fuzzy constraint network.

After applying the \textsc{sequential}\_\textsc{consistency} procedure to an S-MFTP any partial solution can be extended to a global solution with the same degree of satisfaction, by using a backtrack-free algorithm, following the ordering \(o\).
Theorem 1 A satisfiable S-MFTP $N_S$, obtained after SEQUENTIAL_CONSISTENCY has been applied, is decomposable under the assignment order $o$.

Proof 1 See Appendix A.

We can define a topology symmetrical to that of the S-MFTP, in which $M_V$ is dense, and $M_T$ has no cycles. Similar procedures to the ones mentioned above are applicable to this topology. However, the S-MFTP topology has been the most useful in real applications, as often the temporal information is the most relevant in the description of a pattern.

The global pattern presented in Fig. 2 contains a single cycle on its $M_D$ network which violates the S-MFTP topology: the one created by the constraints $D_{01}^1, D_{02}^1$ and $D_{12}^1$. By erasing either $D_{01}^1$ or $D_{02}^1$ the resulting MFTP would be compliant with the S-MFTP topology. Even though this would cause some information about the pattern to be lost, it is unlikely that this information would be essential for the pattern representation, as the other two remaining constraints will still generate an induced constraint which would probably be similar to the erased one. For instance, if $D_{02}^1$ were erased, the absolute value of $X_2^1$ would still be limited by the induced constraint $D_{01}^1 \oplus D_{12}^1$, which does not need to be equal to $D_{02}^1$, but it would likely be close to the erased one.

If a pattern does not follow the S-MFTP topology we can proceed in two different ways. We can obtain path consistency on the networks $M_T$ and $M_V$ and obtain section consistency for the network $M$, thus achieving certain levels of local consistency in the pattern, and then proceed to the pattern recognition, though the network may still contain inconsistencies. The second alternative is to erase those constraints that prevent the pattern from following the S-MFTP topology, and then apply the SEQUENTIAL_CONSISTENCY procedure, to obtain a consistent network. If the constraint/constraints which need to be erased cause an excessive loss of information
in the pattern definition the first option is more attractive. If not, the second one may be considered.

6.2 Hierarchical consistency

The expert tends to describe complex patterns using multiple levels of abstraction. Each new level of abstraction may add new information that refines the information contained in previous levels. However, the consistency analysis procedure we have provided does not tackle each abstraction level independently, rather it merges information coming from different levels. Thus, between each pair of significant points it will supply the same constraint throughout the multiple levels of the hierarchy: the most precise being obtained from a combination of all the induced constraints, i.e., features of the pattern definition that have been incorporated by the expert in higher levels of the hierarchy of abstraction could modify the definition of the findings belonging to lower levels. As a consequence, after analyzing the consistency of the global pattern the findings which make it up may not be equivalent to the original ones that the experts uses to build the hierarchical interpretation of the system behaviour. This would preclude constructing the detection of the pattern just as it was described by the expert, as well as giving explanations about the detection, and sharing subpatterns among different patterns.

For instance, if we analyze the consistency of $\mathcal{M}^{IF}$, the constraints $L_{12}^{23}$ and $L_{24}^{42}$ when propagated to obtain path consistency over the temporal subnetwork will tighten $L_{12}^{4}$, i.e., the maximum duration of $\mathcal{M}^{P4}$ (see Fig. 2). When considered in isolation, or when integrated only in $\mathcal{M}^{MF_2}$, $\mathcal{M}^{P4}$ may last up to two minutes, but when it is integrated in $\mathcal{M}^{IF}$ it must last no more than approximately 30 seconds. As a consequence, the network obtained after analyzing the consistency of $\mathcal{M}^{IF}$ with the
The SEQUENTIAL\_CONSISTENCY procedure does not make it possible to identify all the occurrences of $M^{MF_2}$ and $M^{P_4}$, but only the ones which are suitable for belonging to $M^{IF}$.

We require new consistency analysis procedures which respect the abstraction hierarchy contained in the expert’s description. In this way, the aim is to obtain a consistent version of the findings described by the expert in each abstraction level by incorporating only the information which has been provided in that level and the inferior ones. This is the aim of the HIERARCHICAL\_CONSISTENCY procedure (see Fig. 6). This procedure runs through the abstraction hierarchy of $M$ until it reaches the lowest level, and then, by means of the SEQUENTIAL\_CONSISTENCY procedure, it processes each one of the MFTPs that make up this level. After processing each MFTP the information obtained is incorporated into the next abstraction level. It then goes up to the next level, processes this level, and goes up to the next level, until the global pattern is resolved. We thus avoid the propagation of constraints from higher levels of abstraction to lower ones, i.e., in the opposite direction to the pattern description.

7 Pattern recognition

The ultimate aim of the MFTP model is to identify a pattern $M$ over a set of parameters $\mathcal{P}$, which represent the temporal evolution of a physical system $S$, automatically generating information organized into a hierarchy of levels of abstraction.

Imitating human experts, we divide the recognition task into as many stages as there are levels of abstraction given by the composition of findings. The successive abstraction levels act as successive filters applied to the set of parameters $\mathcal{P}$, which means that a pattern can be identified from the set of findings that make it up. In the
lowest level is the temporal evolution of the system \( P \), and in the highest the global pattern \( M \). Figure 7 shows a recursive procedure which runs through the hierarchy of the pattern until it reaches the lowest level MFTPs; it resolves them and then goes up to the next level of abstraction and resolves the MFTPs there, by assembling solutions found for the MFTPs in lower levels. The procedure comes to an end when solutions for the global pattern are obtained.

For example, in order to recognize the “irreversible fault” pattern, firstly we search for solutions for the subMFTPs that make it up: \( M^{MF_1} \) and \( M^{MF_2} \); and we then attempt to assemble the solutions encountered in order to obtain solutions for \( M^{IF} \). In order to match both malfunction patterns we in turn start by searching for solutions for \( M^{P_1} \) and \( M^{P_2} \), and for \( M^{P_3} \) and \( M^{P_4} \), respectively, and we then attempt to assemble the solutions encountered in order to obtain solutions for \( M^{MF_1} \) and \( M^{MF_2} \), respectively.

The solutions to the MFTPs are constructed incrementally, by exploring a search tree. Let us suppose that the pattern \( M \) has \( a \) nesting levels. We denote as \( M^{l}_{h} = \{ W^{M^{l}_{h}}, X^{M^{l}_{h}}, R^{M^{l}_{h}} \} \) the subMFTP \( h \) in level \( l \), where \( 0 \leq l \leq a \). The 0-level MFTPs are defined over physical parameters and they are not further composed \( M^{0}_{h} = \{ \emptyset, X^{M^{0}_{h}}, R^{M^{0}_{h}} \} \). The a-level MFTP is the pattern \( M \). In order to build a solution \( A^{M^{0}_{h}} \) to \( M^{0}_{h} \) we start from an empty tuple of assignments, and we extend it with an assignment to a new significant point, such that the degree of satisfaction of the extended tuple is greater than or equal to a limit \( c_{inf} \), below which the degree of satisfaction of a solution is not considered to be valid [8]. The degree of satisfaction of a tuple \( A^{r}_{[r]} \) of assignments to \( r \) significant points of \( X^{M^{0}_{h}} \), \( A^{r}_{[r]} = \{ A^{P_{i}}_{p_1}, ..., A^{P_{r}}_{p_r} \} \), is calculated from the degree of satisfaction of \( A^{r-1}_{[r-1]} \), by means of:
\[ \pi_{\mathcal{M}_h^0}(A_{r|}) = \min \{ \pi_{\mathcal{M}_h^0}(A_{r-1}), \min \{ \pi_{R_{i|}^p, A_{r|}}(A_{r|}) \} \} \]  

(3)

where \(R_{i|}^p\) is any constraint between some point to which \(A_{r-1}\) assigns value, and the new point \(X_{r|}^p\). For example, in order to find the pattern \(\mathcal{M}^{P_1}\), we start from an empty tuple of assignments \((A_{P_1}^0 = \{\emptyset\})\), and we add to it an assignment \(A_1^1 = (v_1^1, t_1^1)\) to \(X_1^1\) that satisfies the constraint \(D_{11}^0\), obtaining the tuple \(A_{P_1}^{P_1} = \{A_1^1\}\); we then go on to search for an assignment \(A_2^2 = (v_2^1, t_2^1)\) to \(X_2^1\) that satisfies the set of constraints \(\{D_{02}^1, L_{12}^1, D_{12}^1, M_{12}^1\}\), obtaining the tuple \(A_{P_1}^{P_1} = \{A_1^1, A_2^2\}\); finally we search for an assignment \(A_3^3 = (v_3^1, t_3^1)\) to \(X_3^1\) that satisfies \(\{L_{23}^1, D_{23}^1, M_{23}^1\}\) in order to obtain a solution \(A_{P_1}^{P_1} = \{A_1^1, A_2^2, A_3^3\}\). The degree of satisfaction of the solution \(A_{P_1}^{P_1}, \pi_{\mathcal{M}_h^0}(A_{P_1}^{P_1})\), represents the degree of compatibility of a fragment of the evolution of the system with the pattern represented by \(\mathcal{M}_h^0\). If in any of these steps it is not possible to find an assignment that satisfies the constraints that apply over it, we backtrack to the last significant point to which a value has been assigned and we attempt to carry out another assignment. The same procedure is followed for the recognition of \(\mathcal{M}^{P_2}, \mathcal{M}^{P_3}\) and \(\mathcal{M}^{P_4}\).

The next step is to search for solutions to those 1-level MFTPs \(\mathcal{M}_h^1 = \{W_{\mathcal{M}_h^1}, X_{\mathcal{M}_h^1}, R_{\mathcal{M}_h^1}\}\) for which all subMFTPs \(\mathcal{M}_h^0 \in W_{\mathcal{M}_h^1}\) have been resolved. We start from an empty tuple of assignments, and we extend it by assembling those solutions that have previously been found for their subMFTPs, and by adding assignments to all the significant points belonging to \(X_{\mathcal{M}_h^1}\). We obtain the degree of satisfaction for the set of assignments \(\mathcal{M}_{h}^{M_{h}^1}\) involved in the recognition of \(\mathcal{M}_h^1\) by means of:

\[ \pi_{\mathcal{M}_h^1}(A_{M_{h}^1}) = \min \{ \min \{ \pi_{\mathcal{M}_h^0}(A_{M_{h}^0}) \} \}, \min \{ \pi_{R_{i|}^p, A_{r|}}(A_{r|}) \} \} \]  

(4)
where the first term gathers the solutions to all the subMFTPs \( \mathcal{M}_h \in \mathcal{W}^{\mathcal{M}_h} \), and the second term calculates the degree of satisfaction of the different constraints incorporated in \( \mathcal{M}_h \). This procedure is repeated in each level of the hierarchy of abstraction levels, and it finishes when the global pattern \( \mathcal{M} \) is resolved. The degree of satisfaction of the global solution \( A, \pi_{\mathcal{M}}(A) \), represents the degree of compatibility of a fragment of the evolution of the system with the pattern represented by \( \mathcal{M} \).

For example, in order to assemble assignments for \( \mathcal{M}^{MF_1} \), we start from an empty tuple of assignments \( (A^{MF}_0) = \{ \varnothing \} \), and we add to it a solution \( A^{P_1} \) to \( \mathcal{M}^{P_1} \) \((A^{MF}_3) = \{ A_1, A_2, A_3 \})\). We then search for a solution \( A^{P_2} \) to \( \mathcal{M}^{P_2} \) that satisfies the constraint \( \{ L_{12}^1 \} \), assembling a tuple of solutions \( (A^{MF}_7) = A^{MF} = \{ A^{P_1}, A^{P_2} \} = \{ A_1^1, A_2^1, A_3^1 \} \) and obtaining a final solution for \( \mathcal{M}^{MF_1} \). If, during the search, no extension to the current solution is found that satisfies the constraints of the network, we backtrack to the previous valid solution, and we attempt to assemble a different solution. \( \mathcal{M}^{MF_2} \) is resolved in a similar way. We then start to search for solutions for \( \mathcal{M}^{IF} \), starting from an empty tuple of assignments \( (A^{IF}_0) = \{ \varnothing \} \); then we add a solution to \( \mathcal{M}^{MF_1} \) \((A^{MF}_7) = \{ A^{MF}_1 \})\) and we extend it with a solution to \( \mathcal{M}^{MF_2} \) that satisfies the constraints \( \{ L_{12}^3, L_{23}^3 \} \), obtaining a solution \( (A^{IF}_5) = A^{IF} = \{ A^{MF_1}, A^{MF_2} \} \) to \( \mathcal{M}^{IF} \). Again, if such extension is not possible we backtrack and try another solution for \( \mathcal{M}^{MF_1} \).

Fig. 8 shows the procedure BUILD\_SOLUTION which searches for solutions for an MFTP by extending a locally valid tuple of assignments. \( r \) is the number of significant points which have been assigned value; \( h \) is the number of the subMFTP of the current abstraction level which is going to be assigned; and max is the degree of satisfaction of the solution assembled up to this point. The first time the procedure is called the values of these parameters are 0, 1, and 1, respectively. The list \( S_r \) stores the solutions for
the next subMFTP, or the assignments to the next significant point, which extend the present tuple of assignments with a degree of local consistency higher than $c_{inf}; n^M$ is the number of significant points of $M$; $n^{WV}$ is the number of subMFTP of $M$. After the execution of the RECOGNITION procedure, the solutions found for each pattern $M$, and their degree of satisfaction, remain stored in the corresponding list $H_M$.

7.1 Modelling signal episodes

Until now, the representation of a signal episode has been limited to the events that define the beginning and the end of the interval in which it takes place. Hence, in its current form, the model seems to adequately represent the semantics of expressions of the type “... $P^1$ increases from a low value to approximately fifty units”, where no mention is made of the evolution of the physical parameter during this temporal interval.

Simplifying the representation of a pattern to a set of events over a signal (the significant points) seems to be sufficient when the events are defined over different parameters. Nevertheless, when both events are defined over the same parameter, reducing the representation of an episode to its endpoints is usually not sufficient: different temporal evolutions of a parameter which coincide in the set of events represented by an MFTP, but which differ between them, would be indistinguishable for the model (see Fig. 9).

The natural language allows us to give a set of descriptions in which subtle distinctions are made regarding the way the evolution between two significant points takes place, as in the case of “$P^2$ starts to rise moderately ... until approximately 10 seconds after the sharp fall in $P^1$...”. The MFTP model can be extended to represent information concerning the evolution of a parameter between each two significant points.
using a membership function $\mu_{S_{12}} (A^2_1, A^2_2)$, which defines a fuzzy course (see Fig. 10) within which the temporal evolution of the parameter must remain in order to satisfy the constraint.

Thus, if we understand that "moderately" is satisfied by all samples of the interval which lie between the assignments performed to the significant points $X^2_1$ and $X^2_2$, these samples should verify the change rate represented by $M^2_{12}$ with respect to the assignment carried out on the first significant point. We can model a membership function which forces this behaviour by:

$$
\mu_{S_{12}} (A^2_1, A^2_2) = \min_{t^2_1 \leq t^2_s \leq t^2_2} \max_u \{\mu_{(v^2_s-v^2_1)} \cap M^2_{12} \otimes (t^2_s-t^2_1)(u)\} \quad (5)
$$

where the expression between brackets evaluates the degree of membership of a signal sample $(v^2_s, t^2_s)$ to the fuzzy straight line that is given by the assignment $A^2_1 = (v^2_1, t^2_1)$ and the constraint of $M^2_{12} = "moderately"$. A similar expression can be used to model the evolution between $X^2_2$ and $X^3_2$ (between $X^2_3$ and $X^2_4$) taking the assignment $A^2_2$ ($A^2_3$) as reference, and using the constraint $M^2_{23} = "remains constant" \equiv "approximately 0 units per second"$ ($M^2_{34} = "rate of approximately 4 units per second"$). A more comprehensive study of the fuzzy modelling of natural language sentences describing the evolution of a parameter can be found in [1].

### 7.2 Heuristics for increasing recognition efficiency

With the aim of guaranteeing the completeness of the search, we must find all the possible solutions for each MFTP of each abstraction level, since local solutions of a CSP, even the optimal ones, do not necessarily form part of a global solution, i.e. the global pattern. In order to avoid searching for an excessive amount of solutions we have developed a heuristic which, by making use of the continuity properties of real signals,
aims to obtain a set of solutions that is as representative as possible of each occurrence of a finding. The intuitive idea behind this heuristic is to attempt to obtain a “sampling” of the finding’s occurrence. In order to do so, we search for solutions within a temporal window whose length $L_W$ must be greater than or the same as the maximum temporal extension of the MFTP, to guarantee that no solutions are lost. Each new solution must be better than the best solution up to that point, and we only store those that differ more than $d_{inf}$ from the ones that have already been stored. We define this difference as the Euclidean distance between the vectors of the temporal assignments to the significant points of two different solutions.

When the data from a temporal window have been processed, the window is shifted by a constant interval $S_W$, and the process is repeated. In this way, for each occurrence of a finding we obtain a set of solutions that are scattered around the temporal evolution of the parameters over which the finding is defined. Searching in every window for better solutions than those already found considerably shortens the search space, as it avoids spreading out all the branches that would lead to a poorer solution than the best of those already found. Storing only those solutions that are significantly different limits the search space in the following stages of the recognition, at the same time as it supplies reasonable guarantees of the stored solutions being representative of the temporal evolution of the system.

$S_W$ and $d_{inf}$ act as algorithm control parameters: if they have high values, less time will be needed to perform the recognition, but less solutions will be generated and stored, and there will be a higher probability of finding non-optimal solutions or even of losing occurrences of the pattern. Using lower values results in more time being required for the search, but it is less probable that solutions will be lost or that non-optimal ones will be found. The adjustment of these parameters depends on the
system’s dynamics: for systems that evolve slowly we use higher values, while in
rapidly-evolving systems lower values are recommended. The most precise recognition
is obtained when $S_W = \Delta t$, $\Delta t$ being the sampling period, and $d_{inf}=0$, i.e. all the
solutions found are stored.

Other heuristics that can be applied in the recognition arise from using certain or-
dering criteria in the recognition of the findings that make the pattern up. One strategy
that speeds up the recognition is to first search for uncommon events (an abnormal
value, a distinct morphological feature, etc.), since this allows the constraints that limit
the search space most to be applied first, selecting those signal fragments that are most
promising for the pattern search. The recognition order can be obtained directly from
experts in the domain, as they are aware of which morphological features are less fre-
quent in the evolution of a physical parameter. It is also possible to use a heuristic
which assigns a low probability of occurrence to those findings that have a greater
number of constraints, and in which the constraints are more difficult to be satisfied
(sharper changes in the values of the parameter, more pronounced variations in value,
etc.).

7.3 Real applications of the model

We have built a graphical tool that allows a description of a signal pattern to be pro-
jected into an MFTP, and identified over new signal records: the Tool foR anAlyzing
and disCovering PattErns [21, 22], TRACE (see Fig. 11). TRACE handles each MFTP
as a graph in which each arc represents a set of constraints between the significant
points it connects. These constraints can be edited by clicking over the arc that rep-
resents them. The tool provides a set of utilities and wizards that assist the user in
the modelling of signal patterns. It also allows the user to execute the recognition algo-
rithms and presents the results using visual metaphors to represent the different degrees of compatibility.

Through TRACE, the MFTP model is being applied in the domains of mobile robotics and patient monitoring. In the former the model allows a robot to identify a set of landmarks of the environment by means of sonar sensors, by detecting the particular pattern that these landmarks produce on the sensor signals [24]. In the tests carried out, over 90% of the landmarks were detected, in spite of the noise and imprecision that are characteristic of these sensors. In the patient monitoring domain it can be used to generate intelligent alarms from the temporal evolution of physiological parameters [23]. One example of this is the detection of a signal pattern compatible with a diagnosis of pulmonary embolism, since it is defined by the concurrence of a fall in the oxygen level, with a tachycardia and a relative drop in the systolic blood pressure. Fig. 11 presents TRACE showing the identification of the aforementioned pattern over a register obtained during the hemodialysis of a 35-years-old female patient.

Despite the theoretically high computational complexity of the recognition algorithms, the use of heuristics have allowed us to obtain a real-time performance in real applications. On average, it takes four seconds to search for an MFTP made up of two trends defined over two physiological parameters of a patient over a one hour length recording sampled at 4 Hz [23]. These tests were carried out on a Pentium IV running at 2.4 GHz. In the mobile robotics domain, it takes about 20 seconds to search for an MFTP made up of four different findings over the ultrasound data of a Nomad 200 robot obtained along a 25-minute length trajectory [24]. These tests were carried out on a Pentium III running at 800 MHz.

The most demanding performance test we have carried out is the detection of a sinusal rhythm pattern over an electrocardiogram (ECG) recording lasting a little over
10 minutes that contains a total of 1,263 beats. This test was designed as a stress test for TRACE’s implementation of the recognition algorithms of the MFTP model, and not as an exhaustive validation of a medical pattern. The sinusal rhythm pattern is made up of two subMFTPs; the first contains 4 significant points and the second 2. The ECG was sampled at 250 Hz and two different ECG leads, i.e., two different parameters, were involved in the pattern. The detection was carried out in 75 seconds on a Pentium III running at 800 MHz. We obtained 1,129 true positives, 102 true negatives, 4 false positives and 32 false negatives.

8 Discussion and conclusions

The MFTP model enables us to project a description of the evolution of a set of physical parameters onto a computable representation. This representation can be organized into a hierarchy of abstraction levels, in which a pattern can be defined as the aggregation of a set of sub-patterns. This seems to respond naturally to the manner in which human experts organize their interpretation of perceptual information.

The MFTP is based on two formal tools which endow it with its most distinctive properties: on one hand, a representation based on Fuzzy Set Theory for capturing the vagueness and uncertainty that are intrinsic to expert knowledge, and for responding to the variability and imprecision that are habitual in the problems of interpreting physical systems; on the other hand, the use of CSP formalism lends versatility to knowledge description, as there are a number of ways of describing the same pattern, and the propagation of constraints makes it possible to complete the representation.

The model is capable of representing arbitrary morphologies defined over the temporal evolution of a set of parameters, and a rich set of relations between them. This
provides it with more expressive power than other proposals in the bibliography, which often are concerned with the representation of a single parameter [3, 10, 14, 27]. One exception is the work of Lowe et al. [15], concerned with the representation of multivariable patterns composed by a set of linear trends defined over several parameters. Another contribution of the present work is to provide a formal framework capable of organizing a signal processing task into multiple levels of abstraction. Milios et al. [18] proposed a conceptual framework for organizing the operations of abstraction, along with a set of ad hoc techniques for a specific problem but, to the best of our knowledge, the MFTP model is the first formal framework to provide support for it.

The price to pay for this expressive power is the possibility of inconsistent information being incorporated into the pattern definition. Although the problem of analyzing the consistency of the MFTP is NP-hard, certain levels of local consistency -path and section consistency- can be achieved in $O(n^3)$, where $n$ is the number of significant points of the MFTP. Furthermore, simplifications in the topology of the network permit efficient consistency algorithms to be supplied. The S-MFTP topology provides sufficient expressivity in the pattern description for a good number of real applications, as it adapts to the sequential manner of reading the evolution of a parameter. With the S-MFTP topology it is possible to guarantee backtrack-free solutions in $O(n^3)$. It should be noted that the procedures we have developed to check the consistency respect the abstraction hierarchy contained in the pattern’s description, i. e., they avoid the propagation of information from higher levels of abstraction to lower ones.

The MFTP is equipped with pattern-recognition algorithms, which identify the pattern on the basis of recognizing the set of findings that make it up, along the different abstraction levels into which its representation is organized. In spite of the theoretical computational cost of the detection algorithms, their practical use in the domains of
patient supervision and mobile robotics has demonstrated their possible application in real-time problems. The hierarchical representational structure itself divides the original problem up into a number of simpler problems, and the addition of a heuristic to the recognition process means that the signal properties can be exploited to accelerate the detection process. Furthermore, the abstracted information obtained makes it possible to reduce the computational load of intrinsically complex tasks, such as interpretation or diagnosis.

The pattern acquisition is performed with the assistance of a graphical tool, TRACE, which has proved to be an effective solution to the knowledge acquisition bottleneck. The tool is currently being used in two different domains: patient monitoring, and mobile robotics, where highly satisfactory results have been obtained. TRACE has proved to be a viable solution for users to create their own patterns, and identify them on signal recordings without the need for any type of assistance [22].

With regard to future work, we intend to explore how to incorporate contextual information into an MFTP, so that this information changes the definition of the pattern and adapts it to a given context. This may be especially useful in the medical domain, where the same pattern may present considerably different nuances in different patients due to patients’ physiological variability. One possible solution to this problem is to define a set of operators that, on the basis of contextual information, modify, add, and/or delete constraints, and operators that add and/or delete subMFTPs.

Another possible line of future work is to use the MFTP model in semiautomatic learning tasks. A pattern sketch could be obtained from a human expert by means a graphical tool like TRACE. Using a training data set (a set of signals with occurrences of the pattern), the MFTP constraints could be modified, and significant points could be added or deleted, refining the initial sketch pattern so it captures better the nuances
of the pattern occurrences contained in the training data.

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9 Appendix A

Theorem 2 A satisfiable S-MFTP \( N_S \), obtained after SEQUENTIAL\_CONSISTENCY has been applied, is decomposable under the assignment order \( o \).

This proof is based on a similar proof provided for the FTP model [10]. Here we shall just outline it. The theorem could be demonstrated by induction. The proof supplies a procedure for building solutions with a degree of possibility greater than or equal to a previously set value \( \theta \in [0, 1] \). We carry out an initial assignment \( A_{1i}^{P1} \) to the significant point \( X_{1i}^{P1} \). Given that \( L_{0i}^{P1} \) is a normal possibility distribution, it will always be possible to find an assignment \( A_{1i}^{P1} = (v_{1i}^{P1}, t_{1i}^{P1}) \) such that \( \pi^{L_{0i}^{P1}}(t_{1i}^{P1} - t_{0i}^{P1}) \geq \theta \). Let us now suppose that we have \( k-1 \) assignments \( (1 < k \leq n) \) which satisfy all the constraints among them with a degree greater than or equal to \( \theta \): \( \pi^{M_S}(A_{1i}^{P1}, A_{2i}^{P2}, ..., A_{(k-1)i}^{P_{k-1}}) \geq \theta \).

We can see that it is extensible to the point \( X_{ki}^{P_k} \), i.e., there exists an assignment \( A_{ki}^{P_k} = (v_{ki}^{P_k}, t_{ki}^{P_k}) \) which satisfies all constraints on the previously assigned points with a degree of possibility greater than or equal to \( \theta \): \( \pi^{M_S}(A_{1i}^{P1}, A_{2i}^{P2}, ..., A_{(k-1)i}^{P_{k-1}}, A_{ki}^{P_k}) \geq \theta \). When \( k = z \) we have found a solution for \( M_S \). \( \square \)
10 Figures Captions

**Figure 1**: Stages in the application of the MFTP model.

**Figure 2**: Signal pattern which organizes its information in three levels of abstraction: the first is made up of four morphological findings; in the second the findings are grouped in pairs to obtain the patterns $M^{MF_1}$ and $M^{MF_2}$, which are combined to obtain the global pattern, $M^{IF}$.

**Figure 3**: Topology of the S-MFTP. Dotted arrows represent all the increment constraints allowed by the topology. The quaternary slope constraints are represented by x’s with four arrows. The topology permits any temporal extension constraint. Light grey numbered arrows show a backtrack-free assignment order.

**Figure 4**: Procedure for obtaining sequential consistency.

**Figure 5**: Procedure for obtaining path consistency in $M_T$.

**Figure 6**: Procedure for obtaining hierarchical consistency.

**Figure 7**: Procedure that solves an MFTP hierarchically.

**Figure 8**: Procedure that assembles the solutions of the MFTPs.

**Figure 9**: Two different evolutions have the same degree of compatibility with an MFTP as their values coincide in the assignments made to both significant points.

**Figure 10**: Graphical representations of the fuzzy trajectories employed to model the evolution of $P^2$.

**Figure 11**: TRACE showing the detection of a pulmonary embolism. In the upper part of the window, the contextual information is represented by icons. Below each parameter, the compatibility between the evolution of the parameter and the morphological finding that is defined over it is shown colour-coded. The global compatibility of a pattern is shown by means of the “Detection” parameter.
References


Figure 1: Stages in the application of the MFTP model.
Figure 2: Signal pattern which organizes its information in three levels of abstraction: the first is made up of four morphological findings; in the second the findings are grouped in pairs to obtain the patterns $\mathcal{M}^{MF_1}$ and $\mathcal{M}^{MF_2}$, which are combined to obtain the irreversible fault pattern, $\mathcal{M}^{IF}$. 
Figure 3: Topology of the S-MFTP. Dotted arrows represent all the increment constraints allowed by the topology. The quaternary slope constraints are represented by x’s with four arrows. The topology permits any temporal extension constraint. Light grey numbered arrows show a backtrack-free assignment order.
procedure SEQUENTIAL\_CONSISTENCY($M_S$);
begin
    $Q \leftarrow \{(p, q, r, i, j, k), 1 \leq p, q, r \leq z, 0 \leq i \leq n^p, 0 \leq j \leq n^q, 0 \leq k \leq n^r\}$;
    for each $((p, i), 1 \leq p \leq z, 0 \leq i < n^p)$
        $L^p_{i+1} = L^p_{i+1} \cap (D^p_{i+1} \otimes M^p_{i+1})$;
    while ($Q \neq \emptyset$) do
        begin
            select and delete a path $(p, q, r, i, j, k)$ from $Q$;
            if (REVISE\_PATH($p, q, r, i, j, k$))
                $Q \leftarrow Q \cup RELATED\_PATHS(p, q, r, i, j, k)$;
        end while;
    for each $((p, i), 1 \leq p \leq z, 0 \leq i < n^p)$
    begin
        $D^p_{i+1} = D^p_{i+1} \cap (M^p_{i+1} \otimes L^p_{i+1})$;
        $M^p_{i+1} = M^p_{i+1} \cap (D^p_{i+1} \otimes L^p_{i+1})$;
    end for;
    return($N_S \equiv M_S$);
end procedure;

Figure 4: Procedure for obtaining sequential consistency.
procedure REVISE_PATH(p,q,i,j,k);
begin
    TEMP_L = \( L_{ij}^{pq} \cap (L_{ik}^{pr} \oplus L_{kj}^{rq}) \);
    if (TEMP_L = \( L_{ij}^{pq} \))
        return(FALSE);
    else
        \( L_{ij}^{pq} = TEMP_L \);
        return(TRUE);
end if;
end procedure;

Figure 5: Procedure for obtaining path consistency in \( M_T \).
procedure HIERARCHICAL\_CONSISTENCY(\(\mathcal{M}\));
begin
  if (\(\mathcal{W}^{\mathcal{M}} \neq \emptyset\)) then
    begin
      for each (\(\mathcal{M}_k \in \mathcal{W}^{\mathcal{M}}\))
        begin
          \(\mathcal{M}_k \leftarrow \text{HIERARCHICAL\_CONSISTENCY}(\mathcal{M}_k)\);
          \(\mathcal{M} \leftarrow \mathcal{M} \cap \mathcal{M}_k\);
        end;
      end if;
  \(\mathcal{N} \leftarrow \text{SEQUENTIAL\_CONSISTENCY}(\mathcal{M})\);
return \(\mathcal{N}\);
end procedure;

Figure 6: Procedure for obtaining hierarchical consistency.
procedure RECOGNITION(\(\mathcal{M}\));
\hspace{1em}
if (\(V^\mathcal{M} \neq \emptyset\)) then
\hspace{2em}
for each (\(\mathcal{M}_h \in W^\mathcal{M}\))
\hspace{3em}
RECOGNITION(\(\mathcal{M}_h\));
\hspace{2em}
BUILD\_SOLUTION(\(\mathcal{M}, 0, 1, 1\));
\hspace{1em}
end procedure;

Figure 7: Procedure that solves an MFTP hierarchically.
procedure BUILD_SOLUTION($M$, $r$, $h$, $max$);
if ($r = n^M$) then $H_M ← (A_{[i]}, max)$;
if ($h ≤ n^W$) then
begin
    $S_r ← \{A_{M_h} ∈ H_{M_h} \text{ such that } π_M(A_{[i]} ∪ A_{M_h}) > c_{inf}\}$;
    while ($S_r ≠ ∅$) do
        begin
            take and delete $A_{M_h}$ from $S_r$;
            $A_{[i]} = A_{[i]} ∪ A_{M_h}$;
            $maxx = \min\{max, π_M(A_{[i]}))\}$;
            $h = h + 1$;
            BUILD_SOLUTION($M$, $r + n^M_h$, $h$, $max$);
        end while;
end if
else
begin
    $S_r ← \{A_{p_{r+1}} ∈ P_{p_{r+1}} \text{ such that } π_M(A_{[i]} ∪ A_{p_{r+1}}) > c_{inf}\}$;
    while ($S_r ≠ ∅$) do
        begin
            take and delete $A_{p_{r+1}}$ from $S_r$;
            $A_{[r+1]} = A_{[r]} ∪ A_{p_{r+1}}$;
            $maxx = \min\{max, π_M(A_{[r+1]}))\}$;
            BUILD_SOLUTION($M$, $r + 1, h$, $max$);
        end while;
end if
end procedure;

Figure 8: Procedure that assembles the solutions of the MFTPs.
Figure 9: Two different evolutions have the same degree of compatibility with an MFTP as their values coincide in the assignations made to both significant points.
Figure 10: Graphical representations of the fuzzy trajectories employed to model the evolution of $P^2$. 
Figure 11: TRACE showing the detection of a pulmonary embolism. In the upper part of the window, the contextual information is represented by icons. Below each parameter, the compatibility between the evolution of the parameter and the morphological finding that is defined over it is shown colour-coded. The global compatibility of a pattern is shown by means of the “Detection” parameter.